

Determination of the effective strong coupling constant $\alpha_{s,g_1}(Q^2)$ from CLAS spin structure function data

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Abstract

We present a new extraction of the effective strong coupling constant $\alpha_{s,g_1}(Q^2)$. The result agrees with a previous determination and extends the measurement of the low and high Q^2 behavior of $\alpha_{s,g_1}(Q^2)$ that was previously deduced from sum rules. In particular, it experimentally verifies the lack of Q^2 -dependence of $\alpha_{s,g_1}(Q^2)$ in the low Q^2 limit. This fact is necessary for application of the AdS/CFT correspondence to QCD calculations. We provide a parameterization of $\alpha_{s,g_1}(Q^2)$ that can equivalently be used to parameterize the Q^2 -dependence of the generalized Gerasimov-Drell-Hearn and Bjorken sums.

PACS numbers: 12.38.Qk, 11.55.Hx

In Quantum Chromodynamics (QCD), the gauge theory of the strong force, the magnitude of the coupling is given by the running coupling constant α_s . While α_s is well defined within perturbative QCD at large Q^2 [1], these calculations lead to an infinite coupling at large distances. However, many calculations, including lattice QCD or solving Dyson-Schwinger equations, indicate that α_s remains finite, see e.g. [2] for a review. Most of these theoretical results also predict that α_s loses its scale dependence at large distances (“freezing” of α_s) although there is still no firm consensus, see e.g. the new calculations [3] and [4] (the first result indicates that breaking of chiral symmetry causes α_s to be divergent at large distances while the second confirms previous results of the freezing of α_s). Recently, an effective strong coupling constant, $\alpha_{s,g1}$, has been extracted [5] from experimental data on the Bjorken sum [6]. It is indicative of the freezing of $\alpha_{s,g1}$. When the data are complemented with the generalized Gerasimov-Drell-Hearn (GDH) sum rule [8] and the generalized Bjorken sum rule [7] predictions, the behavior of $\alpha_{s,g1}$ can be established at any distance. Although the connection among the various theoretical techniques used to compute α_s at large distances is unclear, most results exhibit analog behavior and order of magnitude. Likewise, the connection between the experimental results reported in [5] and theoretical techniques is not fully known but they display intriguing similarities. Remarkably, the Q^2 behavior of these calculations and of the data agree. In this paper, we present new results for $\alpha_{s,g1}$ over an extended Q^2 range. We then propose a parameterization of $\alpha_{s,g1}$ and finally discuss briefly the consequences of the behavior seen for $\alpha_{s,g1}$.

We have extracted new data points on $\alpha_{s,g1}$ following the procedure described in [5] and based on the theoretical works of [9, 10]. The advantages and limitations of such extraction are discussed in [5] and summarized here. Effective strong coupling constants are defined using first order pQCD equations. As a consequence, they are process dependent. However, because they can be related to each other using commensurate scale relations [10], they can meaningfully be used for QCD predictions. The definition of effective strong coupling constants using first order pQCD equations makes them renormalization scheme and gauge independent and free of divergence at low Q^2 . They also are analytic when crossing quark mass thresholds. Using the generalized Bjorken sum rule is particularly suited to define the effective strong coupling constant $\alpha_{s,g1}$. Its simplicity makes theoretical calculations easier and it is constrained at both large and low Q^2 by well established sum rules. Furthermore, there is a large amount of data available. A present limitation of the use of effective strong

coupling constants is that the connection between various theoretical calculations of α_s at low Q^2 is so far unclear due to the different approximations used. The relation between theoretical results and experimental extractions is also unclear. In ref. [5], we nonetheless compared all these quantities in order to shed light on possible similarities. We will repeat the comparison here keeping in mind that the same precautions apply.

The new data used to extract α_{s,g_1} were taken with the CLAS spectrometer [11] in Hall B at Jefferson Lab (JLab), using a polarized electron beam with energies ranging from 1 to 6 GeV. The data are reported in [13] and were used to form the Bjorken sum $\Gamma_1^{p-n}(Q^2)$ in a Q^2 -range from 0.06 to 2.92 GeV² [14]. Here, Q^2 is the square of the four-momentum transfered from the electron to the target. Apart from the extended Q^2 -coverage, one notable difference between these data and those of ref. [6] is that the neutron information originates from the longitudinally polarized deuteron target of CLAS while the previous data resulted from the longitudinally and transversally polarized ³He target of JLab's Hall A [12]. The effective coupling α_{s,g_1} is defined by the Bjorken sum rule expressed at first order in pQCD and at leading twist. This leads to the relation:

$$\alpha_{s,g_1} = \pi \left(1 - \frac{6\Gamma_1^{p-n}}{g_A} \right) \quad (1)$$

where g_A is the nucleon axial charge. We used Eq. 1 to extract $\alpha_{s,g_1}/\pi$. The results are shown in Fig. 1. The inner error bars represent the statistical uncertainties whereas the outer ones are the quadratic sum of the statistical and systematic uncertainties. Also plotted in the figure are the first data on α_{s,g_1} from [5] and from the world data of the Bjorken sum evaluated at $\langle Q^2 \rangle = 5$ GeV², α_{s,F_3} from the Gross-Llewellyn Smith (GLS) sum rule [17] measured by the CCFR collaboration [18], and $\alpha_{s,\tau}$ [19]. See [5] for details. The behavior of α_{s,g_1} is given near $Q^2 = 0$ by the generalized GDH sum rule and at large Q^2 , where higher twist effects are negligible, by the Bjorken sum rule generalized to account for pQCD radiative corrections. These predictions are shown by the dashed line and the band, respectively, but they were not used in our analysis. The width of the band is due to the uncertainty on Λ_{QCD} .

The values for α_{s,g_1} from the new data are in good agreement with the previous JLab data. While the previous data were suggestive, the freezing of α_{s,g_1} at low Q^2 is now unambiguous and in good agreement with the GDH sum prediction. At larger Q^2 , the new data agree with the world data and the results from the Bjorken sum rule at leading twist.

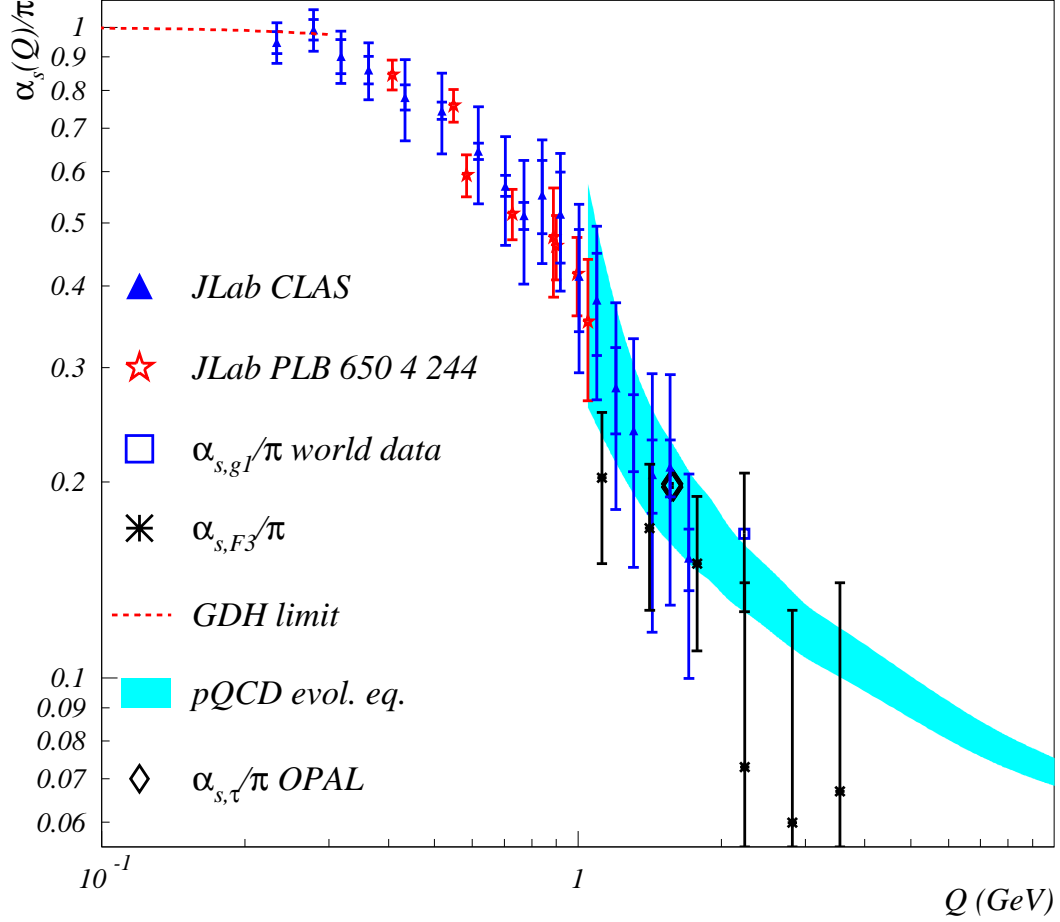


FIG. 1: (color online) $\alpha_{s,g1}(Q)/\pi$ obtained from JLab (triangles and open stars) and world (open square) data on the Bjorken sum. Also shown are $\alpha_{s,\tau}(Q)/\pi$ from OPAL data, the GLS sum result from the CCFR collaboration (stars) and $\alpha_{s,g1}(Q)/\pi$ from the Bjorken (band) and GDH (dashed line) sum rules.

We fit the data using a functional form that resembles the pQCD evolution equation for α_s , with an additional term $m_g(Q)$ that prevents $\alpha_{s,g1}^{fit}$ from diverging when $Q^2 \rightarrow \Lambda^2$ and another term $n(Q)$ that forces $\alpha_{s,g1}^{fit}$ to π when $Q^2 \rightarrow 0$. Note that the latter constraint is a consequence of both the generalized GDH and Bjorken sum rules [5]. Our fit form is:

$$\alpha_{s,g1}^{fit} = \frac{\gamma n(Q)}{\log\left(\frac{Q^2 + m_g^2(Q)}{\Lambda^2}\right)} \quad (2)$$

where $\gamma = 4/\beta_0 = 12/(33 - 8)$, $n(Q) = \pi(1 + [\frac{\gamma}{\log(m^2/\Lambda^2)(1+Q/\Lambda)-\gamma} + (bQ)^c]^{-1})$ and $m_g(Q) =$

$(m/(1 + (aQ)^d))$. The fit is constrained by the data, the GDH and Bjorken sum rules at intermediate, low and large Q^2 respectively. The values of the parameters minimizing the χ^2 are: $\Lambda = 0.349 \pm 0.009$ GeV, $a = 3.008 \pm 0.081$ GeV $^{-1}$, $b = 1.425 \pm 0.032$ GeV $^{-1}$, $c = 0.908 \pm 0.025$, $m = 1.204 \pm 0.018$ GeV, $d = 0.840 \pm 0.051$ for a minimal reduced χ^2 of 0.84. The inclusion of the systematic uncertainties in the fit explains why the reduced χ^2 is smaller than 1. The term $m_g(Q)$ has been interpreted within some of the Schwinger-Dyson calculations as an effective gluon mass [21]. Eqs. 2 and 1 can also be used to parameterize the generalized Bjorken and GDH sums.

The fit result is shown in Fig 2. We also include some of the theoretical calculations (Lattice results and curves labeled Cornwall, Bloch *et al.* and Fischer *et al.*) and phenomenological model predictions (Godfrey-Isgur, Bhagwat *et al.* and Maris-Tandy) on α_s . Finally, we show the $\alpha_{s,g1}$ formed using a phenomenological model of polarized lepton scattering off polarized nucleons (Burkert-Ioffe). These calculations are discussed in [5]. The magnitude of the Godfrey-Isgur and Cornwall results agrees with the estimate of the average value of α_s using magnetic and color-magnetic spin-spin interactions [22]. We emphasize that the relation between these results is not fully known and that they should be considered as indications of the behavior of α_s rather than strict predictions.

The data show that $\alpha_{s,g1}$ loses its Q^2 -dependence both at large and small Q^2 . The Q^2 -scaling at large Q^2 is long known and is the manifestation of the asymptotic freedom of QCD [30]. The absence of Q^2 -dependence at low Q^2 has been conjectured and observed by many calculations but this is the first experimental evidence. This lack of scale dependence (conformal behavior) at low Q^2 shows that conformal field theories might be applicable to study the properties of hadrons. In particular the AdS/CFT correspondence [31] between strongly coupled gauge fields and weakly coupled string states can be used. This opens promising opportunities for calculations in the non-perturbative regime of QCD [32, 33, 34].

Finally, it is noteworthy that conformal behavior is broken in the Q^2 -range between ≈ 0.7 to a few GeV 2 . This domain is the transition region between the fundamental degrees of freedom of QCD (partons) to its effective ones (hadrons).

To summarize, we have used new JLab data on the Bjorken sum to form the effective strong coupling constant $\alpha_{s,g1}$. The Q^2 -range is extended by factors of 3 in both the small and large Q^2 sides compared to results previously reported. The results are in good agreement with sum rule predictions and show for the first time unambiguously that $\alpha_{s,g1}$ loses its

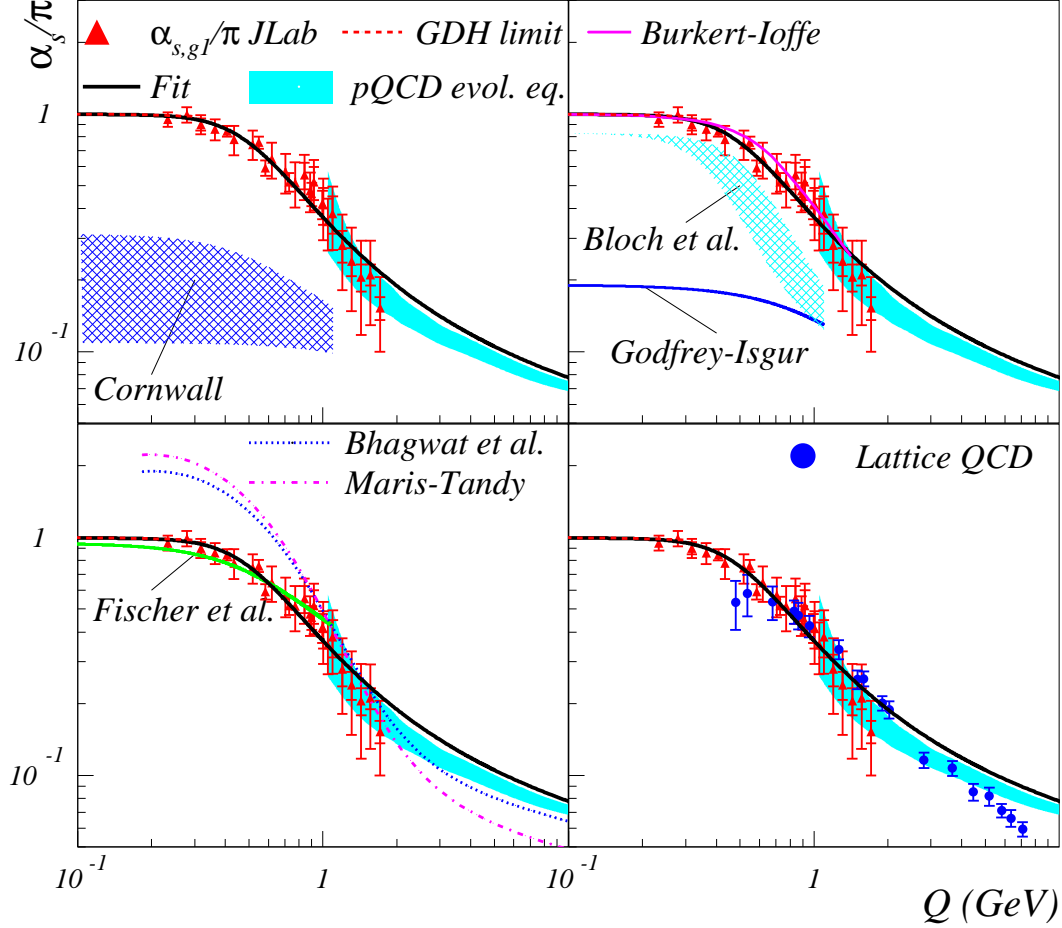


FIG. 2: (color online) The effective coupling constant $\alpha_{s,g1}$ extracted from JLab data, from sum rules, and from the phenomenological model of Burkert and Ioffe [20]. The black curve is the result of the fit discussed in the text. The calculations on α_s are: top left panel: Schwinger-Dyson calculations Cornwall [21]; top right panel: Schwinger-Dyson calculations from Bloch *et al.* [24] and α_s used in the quark model of Godfrey-Isgur [27]; bottom left: Schwinger-Dyson calculations from Maris-Tandy [25], Fischer *et al.* [23] and Bhagwat *et al.* [26]; bottom right: Lattice QCD results from Furui and Nakajima [29].

Q^2 dependence at low Q^2 . We provided an analytic form for $\alpha_{s,g1}$ (or equivalently for the generalized GDH and Bjorken sums) based on the pQCD result for α_s and including the sum rule constraints at small and large Q^2 . It appears that strong interaction is approximately conformal in both the large and small Q^2 limits. We remarked that conformal behavior

breaks down when transiting from the fundamental degrees of freedom of QCD (quarks and gluons) to its effective ones (baryons and mesons). Establishing conformal behavior of strong interaction is a basic step in applying the AdS/CFT correspondence to the study of hadronic matter.

We would like to thank S. Brodsky, S. Gardner, W. Melnitchouk, C. Roberts and P. Tandy for helpful discussions. We acknowledge S. Furui and P. Bowman for sending us their lattice results.

This work was supported by the U.S. Department of Energy (DOE). The Jefferson Science Associates (JSA) operates the Thomas Jefferson National Accelerator Facility for the DOE under contract DE-AC05-84ER40150.

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